Demystifying the vertical datum in Canada: A case study in the Mackenzie Delta

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Abstract

Surveyors are currently going through a transition period for the determination of heights. Over the last few hundred years, the sole technique for precise height determination was spirit leveling. Today, surveyors also have the capability of determining accurate heights using space-based technologies (e.g., GPS) in a more efficient and cost-effective manner. While the leveling technique gives surveyors precise heights above mean sea level when measurements are tied to a benchmark, space-based positioning relates heights to a geometric reference surface (ellipsoid) representing the general shape of the Earth. Unfortunately, the latter does not have any physical meaning (i.e., water could flow up-hill). Therefore, a correction is required to relate ellipsoidal heights to the mean sea level. This is done by using a geoid model, which describes the separation between the ellipsoid and geoid. The geoid is the equipotential (level) surface describing mean sea level at rest.

When using either leveling or space-based positioning for height determination, surveyors require a vertical datum (reference surface), which allows a homogeneous height system at the national, continental or global scale. There may be only one practical definition of a vertical datum (mean sea level); however, its realization will vary as a function of the input data used to model it. This is where complications arise for surveyors: each new realization brings along a different, but generally more accurate datum than the previous one. More recently, the complications were compounded when geoid models became routinely available allowing GPS surveyors to measure significant mismatches between the geoid-derived datum and the distorted leveling-defined Canadian Geodetic Vertical Datum of 1928 (CGVD28).

1. Introduction

This report is intended for those who are involved in positioning and geo-referencing for GIS, mapping, and navigation, and are not geodetic experts. It will help clarify the terminology currently used and the different vertical datums that are available to surveyors in the context of Canada and North America. This will be applied to a case study in the region of the Mackenzie Delta in the Northwest Territories (NWT).

Precise positioning and a consistent height system are the basis for a broad spectrum of activities in earth sciences. These activities range from mapping, engineering and dredging to environmental studies and natural hazards; from precision agriculture and forestry to transportation, commerce and navigation; and from mineral exploration and management of natural resources to emergency and disaster preparedness. While the height reference system supports numerous technical applications, it is also implied in many legal documents related to land management and safety such as easement, flood control, boundary demarcation, etc. All these activities depend on a common coordinate reference system through which all types of geo-referenced information can be interrelated and exploited reliably. For heights, the common and practical reference system is the mean sea level (MSL). Actually, it is the geoid to be exact, as we will see later.

Over centuries sea level has been for surveyors the natural reference surface for heights, and it remains today the best understood global reference surface for heights. Even though the overall concept to define topographical heights is simple, the realization of an accurate reference surface is complex. This complexity has increased even more with the advent of Global Navigation Satellite Systems (e.g., Global Positioning System), which require the development of a geoid model to convert heights from a reference ellipsoid to MSL. Currently, surveyors are at the crossroads between traditional leveling and modern spaced-base techniques for height determination and, unfortunately, the two approaches do not always agree within the required precision due to inherent errors in the realization of each datum. This mismatch is even more accentuated when GPS heights must be converted to Canada's official datum, which is a 1928 construct datum that includes several known systematic errors.

It is important to know that MSL is not an equipotential (level) surface, i.e., as land, the oceans have a topography that ranges from approximately –1.8 m to +1.2 m globally (LeGrand et al., 2003). Vertical datums realized by leveling observations and constrained to several tide gauges will not coincide with an equipotential surface (geoid) because MSL at each tide gauge corresponds most probably to a different equipotential surface (elevation). However, if these same observations were constrained to MSL at a single tide gauge and be errorless, the vertical datum realized would coincide with an equipotential surface. Thus, in theory, leveling and geoid modeling can determine a common vertical datum.

With the knowledge that MSL has topography, commonly referred to as Sea Surface Topography (SST), how can we determine accurately where MSL would be in the middle of Saskatchewan? Actually, MSL can only be determined along the coasts and cannot be propagated accurately inland. For example, the height of a benchmark in Saskatchewan could vary by approximately 60 cm depending on whether it is tied by leveling to a tide gauge on the Pacific, Atlantic or Arctic Ocean. On the other hand, equipotential surfaces are continuous surfaces that can be determined accurately at any location by knowing the Earth's gravity field. Furthermore, the geopotential describes precisely the flow of water, which is not necessarily always the case with heights.

Naturally, a vertical datum is only as good as the input data used for its realization. Systematic and random errors in the data are significant reasons for the discrepancy between the different realizations. In the past, these errors could not be observed easily because the leveling network was re-observed, roughly, on a 25-year cycle. Still today, several leveling lines do not have a second observation epoch and some of them date back to the 1920's. However, over the last ten years, progress in geoid theory along with the acquisition of accurate terrestrial, airborne and spaceborne gravity data with high resolution has allowed for the determination of a more accurate geoid model, revealing errors in the leveling datum to surveyors using space geodetic techniques such as GPS for heighting.

In Canada, the Canadian Geodetic Vertical Datum of 1928 (CGVD28) is the official reference surface for heights. As its name indicates, it is a vertical datum that was realized back in 1928, using leveling measurements that existed at the epoch, to connect the east and west coasts in the southern part of the country. For its realization, several systematic errors in leveling observations were either unknown or disregarded (e.g., SST, actual gravity corrections, refraction and rod calibration) creating a distorted national vertical datum for Canada (though very precise for the time). Today, these distortions are clearly noticeable because new scientific adjustments of the primary leveling network now account for most known systematic errors and accurate gravimetric geoid models are readily available.

2. Concepts and Terminology

Demystifying the understanding about the vertical datum in Canada should start with a review of concepts and terminology that will provide a common ground for discussion on the matter. The explanations of the terms may not be rigorous, but they should serve the purpose of this paper.

First, heights need a reference system because it defines the set of rules that will determine how elevation values will be assigned to a point. For example, fixing MSL at six tide gauges is one set of rules for CGVD28. The reference frame is the actual realization of the reference system. For example, assigning elevations to benchmarks using a defined set of rules results in the realization of a unique reference frame for leveling. The development of a geoid model also constitutes a reference frame.

The vertical datum is the reference surface for heights; it corresponds to the zero elevation. It can be any arbitrary surface as long as it remains constant for different users to reproduce the same height value at a common point. However, the vertical datum should preferably represent an equipotential surface (W), i.e., a level surface on which the water is at rest. An infinity of equipotential surfaces surrounds the Earth. These surfaces are usually expressed in units of $m^2\ s^{-2}$ or kGal m, where one kGal represents $1x10^1$ m s^{-2}. The equipotential surface (W_0) that best represents MSL globally is usually referred to as the geoid. However, in practice, the geoid could be any equipotential surface close to MSL. The geometric distance above the geoid and along the plumbline (perpendicular to the equipotential surfaces) is the orthometric height. It is commonly known as the height above mean sea level. Nowadays, the vertical datum could also be the ellipsoid, which is the reference surface for the ellipsoidal heights as derived from GPS.

Water management is an application where precise heights are required. Water flows in the direction of decreasing elevation in response to variations in the Earth's potential. The equipotential surfaces mentioned above are not parallel; they actually converge and are closer to each other at the poles where gravity is larger than at the equator. This convergence makes the geometric distance between two equipotential surfaces shorter in the north than in the south (for the northern hemisphere). Thus, the surface of a lake, which is an equipotential surface, would have a higher orthometric height at its southern end than at its northern end. For this reason, hydrologists work instead with dynamic heights, which are scaled geopotential numbers. A lake surface has a constant dynamic height. The International Great lakes Datum 1985 (IGLD85) uses dynamic heights. In most applications, the relative difference between orthometric and dynamic heights is negligible.

The orthometric height (H) and dynamic height (H^d) can be expressed as:

$$H = \frac{C}{\bar{g}} \tag{1}$$

and

$$H^d = \frac{C}{\gamma_\phi}, \tag{2}$$

respectively. C is the geopotential number, $\bar{g}$ is the mean gravity between the topography and datum along the plumbline and γ_ϕ is a constant representing normal gravity, which is an approximate gravity value derived from a mathematical model. For dynamic heights in North

America, the normal gravity is evaluated on the ellipsoid at latitude 45°. The geopotential number C can be determined from leveling observations by:

$$\Delta C_{ij} = (\Delta H_{ij} + \varepsilon)\frac{g_i + g_j}{2}, \tag{3}$$

where ΔH_{ij} is the measured height difference by leveling technique between points i and j, ε is a correction for systematic errors and g_i and g_j are the observed gravity on the terrain at points i and j, respectively.

Similarly, orthometric height H can be determined from space-based technologies by using the following equation:

$$H = h - N, \tag{4}$$

where h is the ellipsoidal height and N is the geoid height, i.e., the separation between the ellipsoid and geoid. A geoid height can be determined from gravity data by solving Stokes's global integral (Heiskanen and Moritz, 1967):

$$N = \frac{R}{4\pi\gamma_0(\phi)}\int_{\Omega} \Delta g\, S(\psi)\, d\Omega, \tag{5}$$

where Δg are gravity anomalies, $S(\psi)$ is the weight function, R is the Earth mean radius and γ_0 is the normal gravity on the ellipsoid along a latitude (ϕ).

Bathymetry shown on hydrographic charts is referenced to <u>chart datum</u>. It corresponds to the lower low water in order to assure clearance for vessel transit. Water level rarely goes below the chart datum. Chart datum is a local reference surface and does not correspond to a particular equipotential surface. A chart datum is referenced to local physical markers near tide gauges. These markers have known <u>height above the chart datum</u> (*CD*). The separation between the MSL (based on tide gauge measurements) and the chart datum is the $\underline{Z_0}$ value.

There are also <u>normal heights</u>, which are used in several European countries. These heights are similar to orthometric height with the exception that the term in the denominator of Eq. 1 is replaced by the mean normal gravity at the station along the normal, i.e., perpendicular to the ellipsoid. Thus, they do not require the knowledge of the topographical density making them somewhat advantageous over the orthometric heights and easy to transform to dynamic heights. The ellipsoidal heights can be related to the normal heights through the <u>telluroid</u> (quasi-geoid). The telluroid is not an equipotential surface.

Finally, it is also important to distinguish between <u>accuracy</u> and <u>precision</u>. In this report, accuracy indicates the <u>absolute</u> error with respect to the "*true*" reference datum while the precision represents the relative error between two points on the same datum. For example, two geoid heights within a few km could have an accuracy of ±10 cm while their relative precision can be ±1 cm. Also, a CGVD28 height could be said to have an accuracy of ±50 cm with respect to an equipotential surface, but also have an accuracy of ±5 cm with respect to its own reference system.

3. Datums

Basically, there are two techniques to realize a national vertical datum: leveling and geoid modeling. Leveling is the traditional technique for the realization of a datum. It is determined in relation to the topography where benchmarks have known orthometric heights. Geoid modeling is the modern technique, even though geoid concepts date back to F. Gauss (1777-1855). Geoid modeling came to the forefront with the advent of space-based positioning in order to relate ellipsoidal heights to MSL. Geoid models describe the vertical datum in relation to an ellipsoid. In theory, the two techniques determine the same vertical datum. However, in practice, the two datums will be different due to systematic and random errors in the input data of the two techniques. Thus, the objective of all existing realizations of a vertical datum (CGVD28, NAVD88, GSD95, CGG2000) is to represent "MSL" as accurately as possible.

3.1 Leveling datum

Most countries, if not all, still rely on leveling for the realization of their national vertical datum. The main advantage is its very high precision over short distances. On the other hand, it is also laborious, time consuming and prone to accumulation of systematic errors over long distances. The leveling technique consists in measuring a height difference between two graduated rods roughly 100 m apart in a leapfrog fashion (Figure 1). Thus, surveyors literally have to walk across the country to establish reference markers (benchmarks) along main transportation roads. These benchmarks provide physical access to the datum.

The official vertical datum in Canada is determined by leveling and accessible through some 80,000 benchmarks mostly distributed in southern Canada. It is referred to as the Canadian Geodetic Vertical Datum of 1928 (Canon, 1928 and 1935). It is based on an adjustment of leveling measurements made prior to 1928 with constraints to MSL at six tide gauges: two on the Pacific Ocean (Vancouver and Prince-Rupert), three on the Atlantic Ocean (Yarmouth, Halifax and New York City) and one on the St-Lawrence River (Pointe-au-Père, east of Rimouski). These tide gauges constitute the reference system for CGVD28. Since then, all leveling measurements consisting of re-observations or extensions to the network, have been processed following the same procedure and constrained as the 1928 original adjustment. Figure 2 illustrates the coverage of the primary leveling network in Canada. The primary leveling networks for

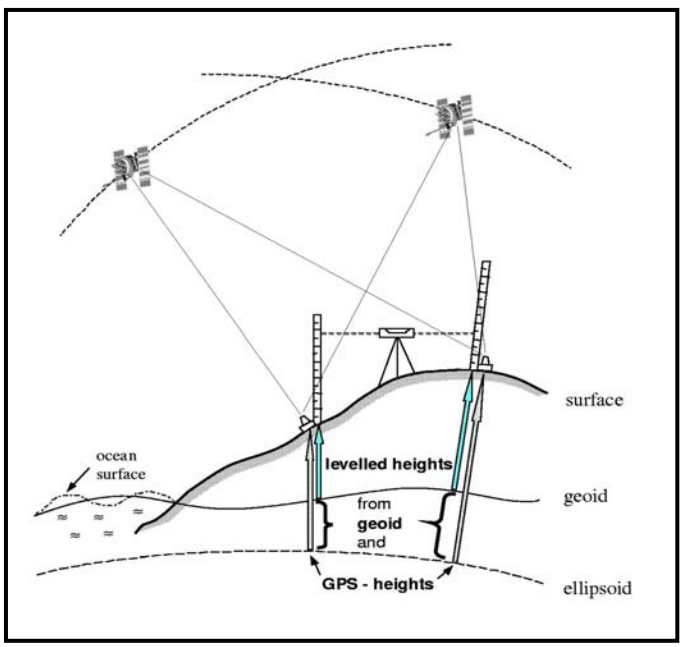

Figure 1: Illustration depicting two techniques (leveling and GPS) for the determination of height differences.

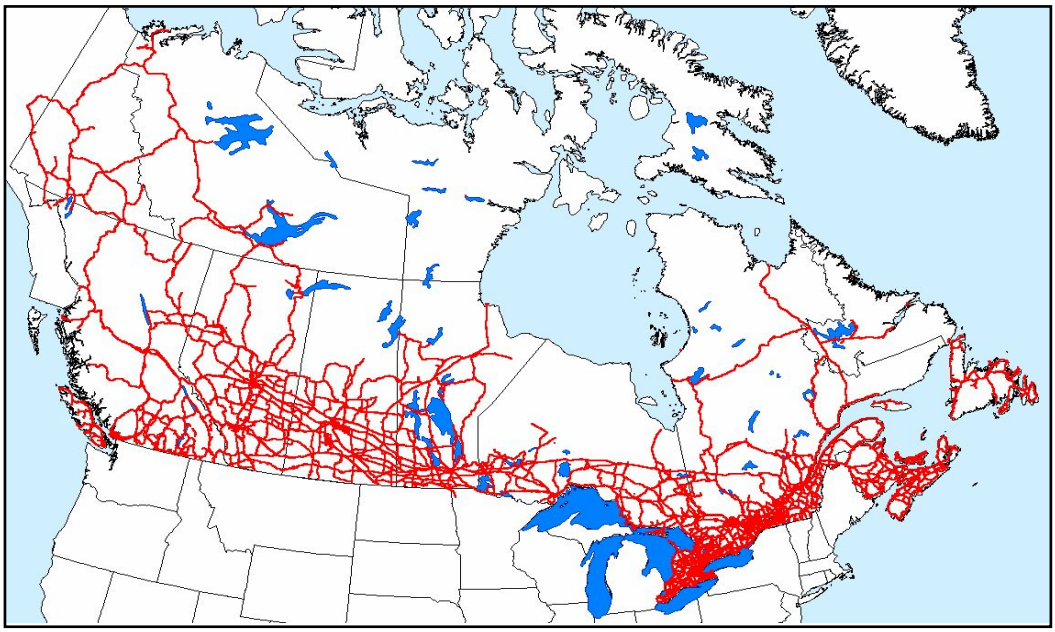

Figure 2: The Canadian primary leveling network

Newfoundland, Prince Edward Island, Anticosti and Vancouver Island belong to CGVD28 even though they are not directly tied to the six tide gauges mentioned above.

The published CGVD28 heights provide a standard reference frame that meets the needs of the majority of users in Canada. However, these heights have not been corrected for certain systematic errors that have become well known. While published values are occasionally corrected for gross errors, neglecting corrections for systematic errors do not adversely affect the precision of published heights within a region. The following describes some of the existing systematic errors:

- CGVD28 heights are computed using approximate gravity values (normal gravity) based upon latitude instead of the actual gravity measurements;
- CGVD28 does not take into consideration that the mean sea level is rising due to the melting of glaciers and ice caps and ocean thermal expansion, and that the land elevation is changing due to the rebound (or subsidence) of the Earth's crust following the last glaciation;
- CGVD28 heights are not corrected for systematic errors due to atmospheric refraction, rod calibration and rod temperature, and the effects of solar and lunar tides on the Earth's geopotential surfaces; and
- CGVD28 does not take into account sea surface topography.

The CGVD28 heights are said to be <u>normal-orthometric heights</u> because they are defined following the concept of orthometric height (Eqs. 1 and 3), but all actual gravity measurements in these equations are replaced by normal gravity. Thus, CGVD28 heights are neither normal nor orthometric heights and CGVD28 coincides neither with the geoid nor the quasi-geoid.

The above-mentioned systematic errors are well known. Geodesists in Canada and the United States had similar vertical datum issues prior to the 1993 adoption of the North American Vertical Datum of 1988 (NAVD88). During the late 1970's and early 1980's, the Canadian and American

geodetic agencies cooperated towards the realization of a new vertical datum that would encompass corrections to most systematic errors. The resulting NAVD88 (Zilkoski, 1986; Zilkoski et al., 1992) is a minimum constraint adjustment of the North American primary leveling networks (Canada, USA and Mexico). The reference system of NAVD88 is the MSL at the tide gauge in Rimouski, Québec.

NAVD88 was not implemented in Canada because of unexplained discrepancies of the order of 1.5 m from east to west coasts (likely due to accumulation of systematic errors) and the slight improvement overall that this new datum would bring. Since then, Canada has continued its data analysis and experimental adjustments of the primary leveling network; however, the efforts were expended mainly for the validation of geoid models in Canada. The latest adjustment of the national primary leveling network in Canada, referred as Nov04 (also a minimum constraint adjustment like NAVD88), indicates a discrepancy of 80 cm between the east and west coasts. Oceanographers estimate that this discrepancy should be around 50 cm. Nov04 represents *true* orthometric heights that can be compared directly with those derived from the combination of ellipsoidal heights and a geoid model.

The actual accuracy of the Nov04 reference frame is difficult to assess. There is still an accumulation of systematic errors in the network of the order of 0.1 mm/km due to old instrumentation and uncertainty regarding corrections applied to the measurements. Furthermore, leveling lines observed at a single epoch may have unknown blunders that may not be detected until leveling is repeated. For example, most leveling throughout the Yukon Territory and Northwest Territories was observed once during the 1970's. The accuracy of Nov04 with respect to the equipotential surface representing MSL in Rimouski can be estimated at the decimeter level while CGVD28 would have an accuracy of several decimeters with respect to the same reference surface. On the other hand, the precision of the leveling is estimated to be better than 4 mm x $(K)^{1/2}$ where K is the distance in km between two benchmarks. Analysis of leveling loops observed after 1980 reveal that it can be as precise as 1.5 mm x $(K)^{1/2}$. These overall precisions are valid as long as benchmarks remain stable.

3.2 Geoid Datum

Geoid modeling is an alternative technique to leveling for the realization of a vertical datum. Canada is currently leading the way in implementing and adopting a geoid model as an official and national vertical datum (Véronneau et al., 2005). Previously, geoid models did not have the required precision and accuracy to be used as a datum. Several regions of Canada did not have gravity measurements and precise theoretical equations were not fully developed at the time. However, the advent of GPS and other space-based techniques and their potential for precise heighting have driven the requirement for improved geoid modeling, leading to the challenging objective of achieving an accuracy of 1 cm across Canada. Today, most of the country and surrounding oceans have gravity measurements with the exception of a few areas: Great Bear Lake, Lake Athabasca, Bay of Fundy and a few small sectors in the Arctic. Furthermore, the theory has been developed to achieve the millimetre accuracy thanks to national and international cooperation between several governmental agencies and academic institutions.

The latest published Canadian geoid model CGG2000 (Véronneau, 2001), while not meeting the accuracy requirement of a new datum, confirms the potential of a gravity-based system as a seamless vertical datum covering all of the Canadian territory and surrounding oceans. Current international progress in satellite gravimetry contributes greatly to improving the accuracy of the geoid model in Canada and will likely enable its adoption as a new datum. The CHAMP and

GRACE gravity missions now allow the cm accuracy for the long wavelengths (~600 km) of the geoid model. While the future GOCE gravity mission (2007) will hopefully bring this same level of accuracy down to the ~100-km wavelengths. The terrestrial gravity data (land, shipborne and airborne surveys) and Digital Elevations Models (DEM) complement well the space-based data by improving the geoid model regionally and locally.

The latest experimental geoid model CGG05 (soon to be published), which is based on early results from the CHAMP and GRACE gravity missions, has a national standard deviation of 14 cm when compared to geoid heights derived from GPS ellipsoidal heights and Nov04 orthometric heights. If a systematic tilt is removed from the leveling data, the standard deviation decreases to 6 cm. This same comparison with CGVD28 indicates a standard deviation of 20 cm. These standard deviations are not entirely representative of the accuracy of CGG05 because they also include errors from the orthometric and ellipsoidal heights.

The main challenge remains to demonstrate the actual accuracy and precision of the geoid models. As for leveling, geoid modeling is not absent of systematic errors. They leak principally from the terrestrial gravity anomalies required in the Stokes integral. Gravity anomalies are differences between measured and normal gravity, corrected for terrain effects. However, the systematic errors in geoid modeling are estimated to be smaller than those from leveling data.

An error propagation model indicates that CGG05 has an average accuracy of ±6 cm nationally. The accuracy is a few cm for the flat regions (e.g., Saskatchewan), but it can reach over the decimeter level in the rough terrain of the Western Cordillera. On the other hand, the precision of the geoid model could be as good as 1 to 3 cm for baselines less than 100 km based on its validation against GPS measurements on benchmarks. The current weakness in the geoid model could be for baselines between 100 km and 600 km where the precision could be less due to accumulation of systematic errors in the terrestrial gravity anomalies.

The current geoid models available are:

Canadian Gravimetric Geoid 2000 (CGG2000):

> Latest published scientific geoid model for North America developed at Natural Resources Canada (Véronneau, 2001). The model represents the separation between the GRS80 ellipsoid and geoid (W_0 = 62636855.8 m^2/s^2) in the International Terrestrial Reference Frame (ITRF).

Canadian Gravimetric Geoid 2005 (CGG05):

> Latest experimental geoid model for North America developed at Natural Resources Canada. This model includes early results from the CHAMP and GRACE space gravimetry missions. CGG05 is the new benchmark for the development of the next generation of models in Canada. The model represents the separation between the GRS80 ellipsoid and geoid (W_0 = 62636856.88 m^2/s^2) in the International Terrestrial Reference Frame (ITRF). This equipotential surface represents a separation of +11 cm with respect to CGG2000 and approximately -30 cm with respect to the mean water level at the tide gauge in Rimouski. The mean water level near Rimouski would be lower than CGG05 datum.

Even though a geopotential surface is independent of the horizontal reference frame, the realization of a geoid model requires a reference frame to relate the geoid to its ellipsoid. It is important to differentiate the NAD83 (CSRS) and ITRF geoid models because the geocentre of

these two frames is approximately two metres apart. This represents a 1.4 m slope across Canada (over ~6000 km) between the NAD83 (CSRS) and ITRF geoid models. On the other hand, the geocentre of the more recent ITRF realizations differs only at the centimetre level. Thus, the difference is negligible for geoid modeling. Furthermore, it is important to know that WGS84 and ITRF are basically the same. The latest version of WGS84 is based on ITRF2000. Finally, transformation software between the different reference frames are available from Geodetic Survey Division of Natural Resources Canada.

3.3 Hybrid datum (Height Transformation)

The hybrid datum is the source of many problems. It is a *band-aid* solution to help surveyors using GPS technologies to tie their surveys to the official vertical datum. The role of the hybrid datum is to convert a datum known only at benchmarks (leveling datum) to a continuous datum by the intermediary of a geoid model. This conversion would be relatively simple if the leveling datum would coincide to an equipotential surface. It would consist of observing GPS on a few benchmarks and evaluating the systematic bias through:

$$h - H - N = \varepsilon \tag{6}$$

where ε is the local bias between the leveling and geoid datums.

The main problems in Canada are that:

- CGVD28 is defined by normal-orthometric heights;
- CGVD28 contains systematic errors;
- Several leveling lines have not been re-observed within the last 30 years increasing the possibility of significant vertical motion at some benchmarks;
- There are no *simple* mathematical functions for transforming a geoid model to CGVD28; and
- CGVD28 benchmarks are poorly distributed geographically, they are mostly located in southern Canada and are sparse or inexistent in northern Canada.

These problems require that the geoid model be distorted to represent CGVD28. These distortions are so significant that Canada refers to this new surface as a Height Transformation to avoid misleading users who might think CGVD28 coincides with the geoid. In the USA, the realization of a hybrid datum is *relatively* easier because they adopted NAVD88 (orthometric heights) and have a leveling network that covers their landmass well. Their hybrid datum is called a geoid (e.g., Geoid99, Geoid03).

While the US National Geodetic Survey observed GPS on benchmarks at some 11,000 sites distributed fairly homogenously throughout the continental US; Geodetic Survey Division of Natural Resources has 2,243 benchmarks co-located with GPS measurements. These GPS stations are part of the Canadian GPS network commonly referred as Supernet version 3.3a (SN33a). The sparse network of GPS on benchmarks in northern Canada makes it difficult to interpolate and extrapolate the geoid model to represent CGVD28. Figure 3 depicts the existing distortion in CGVD28 vis-à-vis the CGG05 geoid model. This figure can be compared with Figure 4, which illustrates the datum difference of the same geoid model with Nov04. Nov04 has mostly an east-west systematic tilt with a few local distortions due to local instability of the benchmarks and to old leveling lines.

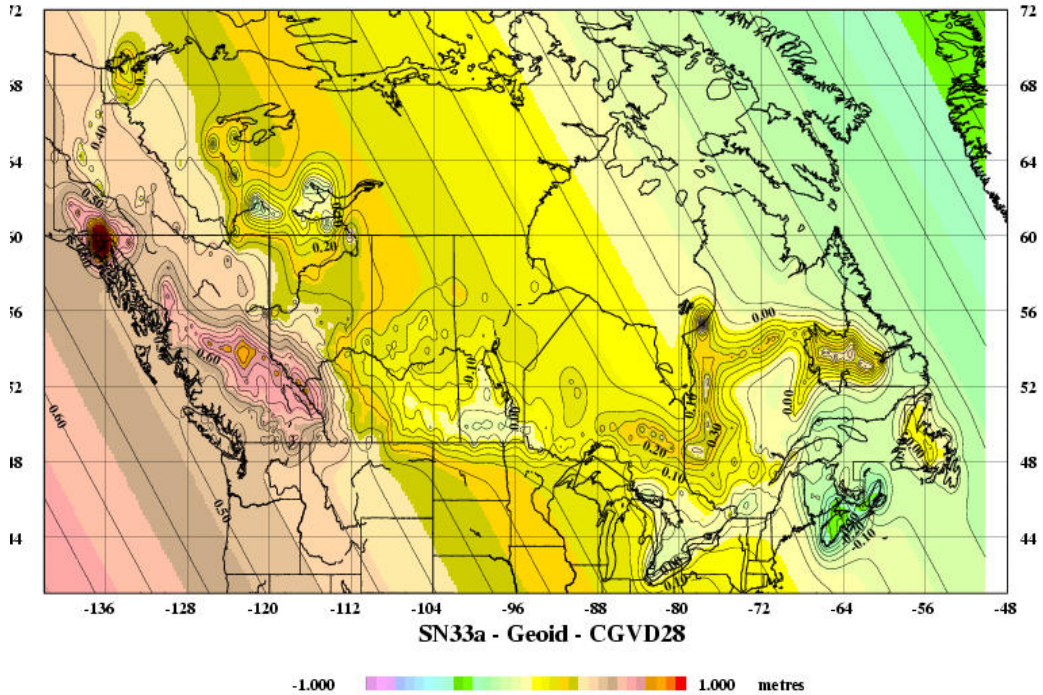

Figure 3: Distortion between CGG05 and CGVD28 (C.I.: 0.05 m)

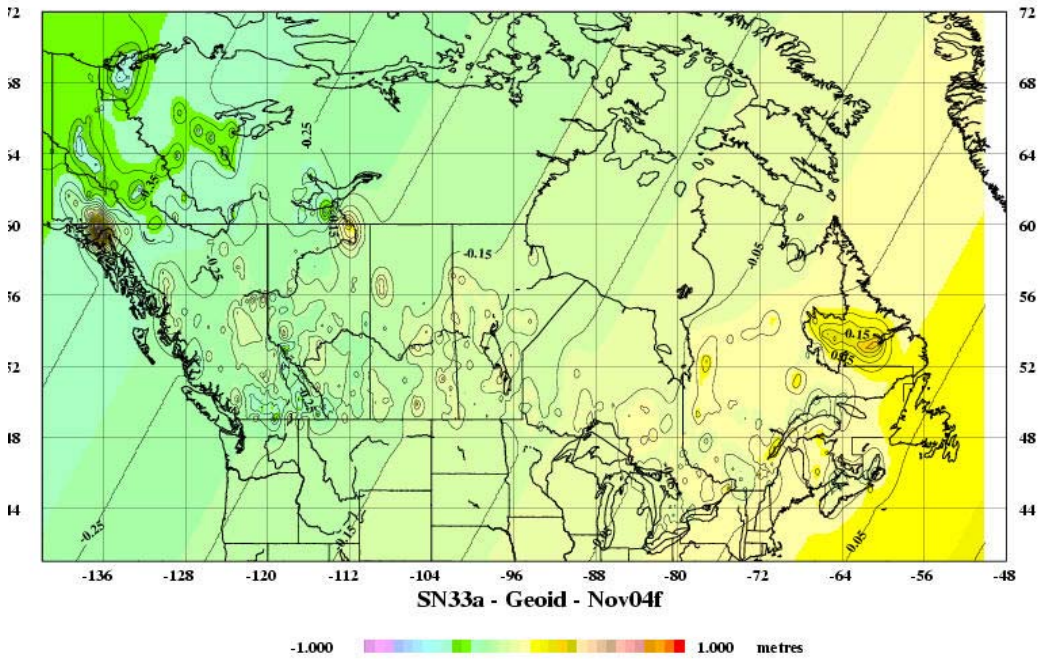

Figure 4: Distortion between CGG05 and Nov04 (C.I.: 0.05 m)

The latest Height Transformation for Canada is HTv2.0 (Véronneau et al., 2001).

HTv2.0:

> Height Transformation version 2.0 is based on CGG2000. A Height Transformation (HT) is a distorted geoid model to represent as accurately as possible the official vertical datum of Canada (CGVD28). A HT represents the separation between the GRS80 ellipsoid and CGVD28 in NAD83 (CSRS) reference frame. The quality of the HT depends on:
>
> 1) The precision of the GPS measurements on benchmarks;
> 2) The distribution of these GPS on benchmarks across the leveling network; and
> 3) The stability of the benchmarks.

The HT is more accurate than a scientific geoid model (e.g., CGG200 and CGG05) with respect to CGVD28, but its precision is less due to the three reasons mentioned above. Furthermore, the HT will reproduce all systematic errors and even possible blunders that are part of CGVD28. The accuracy of the HT can reach a few decimeters in remote regions where leveling is sparse or inexistent.

3.4 Chart datum

This section is not intended to describe the chart datum in detail. It limits itself to explaining how land surveys, done either by leveling or GPS techniques, can be related to the MSL. Fisheries and Oceans Canada (DFO) is responsible for the maintenance of the chart datums. At each tide gauge, DFO establishes a chart datum. It is set to a level where water level rarely goes below it. In general, the Lower Low Water Large Tide (LLWLT) defines the chart datum. Each datum is independent unless leveling or GPS surveys tie the tide gauges together. In this case, the chart datums can be referenced with respect to a vertical datum (benchmarks, geoid or ellipsoid).

At each tide gauge, DFO measures regularly the water level with respect to the chart datum. From these measurements, DFO can establish MSL above the chart datum. The separation between the MSL and the chart datum is the Z_0 value. Furthermore, the stability of the tide gauge is maintained by referencing it to a series of markers in close proximity having heights known above the chart datum. *CD* expresses the height of a reference point above the chart datum. Thus, if the reference point is also known above a vertical datum, the separation between the vertical datum and MSL (Δ_{datum}) can be determined either by

$$\Delta_{datum} = H - CD + Z_0 \tag{7}$$

or

$$\Delta_{datum} = h - N - CD + Z_0. \tag{8}$$

A negative Δ_{datum} would mean that the MSL is below the vertical datum. If the vertical datum corresponds to the geoid (the global MSL), Δ_{datum} would represent the actual sea surface topography.

4. Datums for the Mackenzie Delta

Now, let's consider the datum situation in the Mackenzie Delta, NWT. The Delta region covers an area of approximately 20,000 km^2 at the mouth of the Mackenzie River, next to the Beaufort Sea. A significant portion of the Delta has an elevation of less than a couple of metres. It is a region where floods are common during storm surges and spring freshet. Furthermore, the Delta is currently a region of important economic activity with the development of a gas project and, naturally, environmental issues. Thus, the Delta requires an accurate vertical datum. A decimetre height difference could represent different scenarios of flooded areas. Figure A.1, in appendix, illustrates the relation between the different datums in Tuktoyaktuk.

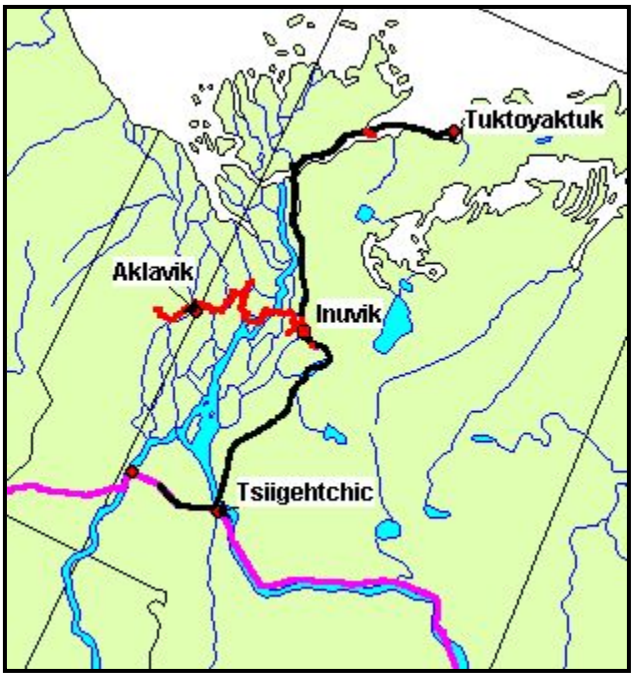

Figure 5: Leveling surveys in the Mackenzie Delta in 1970's (magenta), 1980's (black) and 1990's (red).

During the 1970's, leveling surveys were conducted in the Yukon and NWT to extend CGVD28 into the western Canadian Arctic (Figures 2 and 5). Leveling along the Alaska, Klondike and Dempster Highways and the Mackenzie River were done to tie Whitehorse, Dawson City, Tsiigehtchic (formerly Arctic Red River), Norman Wells, Fort Simpson and Fort Providence to a common reference height reference system. Tuktoyaktuk was tied to Tsiigehtchic by leveling in 1987-1988. The sections south and north of Inuvik were surveyed in the summer and winter times, respectively. The western segment from Inuvik to Aklavik was surveyed in the winter of 1991.

The leveling data between Tsiigehtchic and Tuktoyaktuk was adjusted by constraining the two ends. The southern end was constrained to the CGVD28 height derived from the 1970's survey and the northern end was constrained to MSL at the tide gauge (# 6485) in Tuktoyaktuk. Thus, CGVD28 was made to coincide with MSL in the Delta region. This can be verified by using Eq. 7. Table 1 indicates the separation between CGVD28 and MSL (Δ_{CGVD28}) at some reference markers to tide gauge #6485 in Tuktoyaktuk. Stations 66T9503, 66T9504, 73T9515 and 749151 were leveled in 1987 following first-order standard while station M039008 was tied directly by leveling to station 66T9503 in 2004. No check leveling was conducted to verify stability of the existing local benchmarks. Thus, heights at M039008 could be erroneous if station 66T9503 has moved since 1988.

The results in Table 1 show a separation of approximately 1 cm between CGVD28 and MSL, which can be considered negligible. A negative Δ_{CGVD28} indicates that MSL is slightly below the vertical datum (CGVD28). Furthermore, station 66T9503 appears to be stable because station M039008 has comparable Δ_{CGVD28} to the other stations.

Table 1: Separation between CGVD28 and MSL (Δ_{CGVD28}) at Tuktoyaktuk (Unit: m).

Station	H_{CGVD28}	Tide Gauge #6485		Δ_{CGVD28}
		C.D.	Z_0	
66T9503	2.690	3.064	0.364	-0.010
66T9505	2.690	3.064	0.364	-0.010
73T9515	5.945	6.317	0.364	-0.008
749151	5.671	6.052	0.364	-0.017
M039008	4.739	5.113	0.364	-0.010

However, by constraining the leveling section in Tsiigehtchic and Tuktoyaktuk, does the adjustment create a systematic error? It is unlikely that the heights in Tsiigehtchic are accurate because of the accumulation of systematic errors throughout CGVD28. The comparison of CGVD28 with Nov04, which is a free adjustment north of Tsiigehtchic, indicates a discrepancy that is increasing systematically from –73 cm to –55 cm in the northern direction over almost 300 km of leveling (see Figure A.2). It constitutes an error of 0.6 mm/km in CGVD28. If the local MSL at Tuktoyaktuk would define the vertical datum, CGVD28 heights in Tsiigehtchic should be higher by 18 cm. The large absolute differences come from the fact that Nov04 is a minimum constrained adjustment by holding the height of a station in Rimouski fixed with respect to its local MSL. The separation between Nov04 datum and MSL (Δ_{Nov04}) in Tuktoyaktuk is given in Table 2. If we assume that Nov04 has no accumulation of systematic error, Δ_{Nov04} indicates that the local MSL at Tuktoyaktuk is higher than the local MSL at Rimouski by 53 cm.

Table 2: Separation between Nov04 and MSL (Δ_{Nov04}) at Tuktoyaktuk (Unit: m).

Station	H_{Nov04}	Tide Gauge #6485		Δ_{Nov04}
		C.D.	Z_0	
66T9503	3.232	3.064	0.364	0.532
66T9504	3.232	3.064	0.364	0.532
73T9515	6.484	6.317	0.364	0.531
749151	6.210	6.052	0.364	0.522
M039008	5.281	5.113	0.364	0.532

The hybrid datum HTv2.0 should also coincide with MSL because it is a realization of CGVD28 by definition. However, at the time of its realization, only a single GPS site on a benchmark was available in the Delta making the correction rather weak. The GPS measurements came from an early survey in 1987, when accuracy of the GPS measurements were at the decimeter level. Because the stations are rather sparse, it is difficult to confirm stability of the benchmarks. Figure A.3 shows the location of the few GPS/Leveling stations available for the realization of HTv2.0 in 2001, and the discrepancies between CGG2000 and CGVD28 at those stations. Also, it depicts the 30-cm distortion applied to CGG2000 in the region to create HTv2.0. Naturally, this distortion reduces the precision of the Height Transformation.

It is only in 2004 that an extensive GPS campaign was conducted in the Delta region to validate the precision of the geoid model (Figure A.4). This campaign included observations at station M039008, which is a reference marker to the tide gauge in Tuktoyaktuk. Thus, it is possible to

determine directly the separation between HTv2.0 datum and MSL ($\Delta_{HTv2.0}$) at Tuktoyaktuk by using Eq. 8:

$$\Delta_{HTv2.0} = h_{NAD83} - N_{HTv2.0/NAD83} - CD + Z_0. \quad = -2.767 - -7.164 - 5.113 + 0.364 = -0.352 \text{ m}$$

A 35-cm separation indicates that HTv2.0 is a poor realization of CGVD28 in the region, which is not surprising based on the constraints that were available. Figure A.5 shows discrepancies between HTv2.0 and CGVD28 at the remaining points of the 2004 GPS campaign. A valid height transformation should have residuals (*h-H-N*) near zero with an approximate error of ±2.5 cm or better.

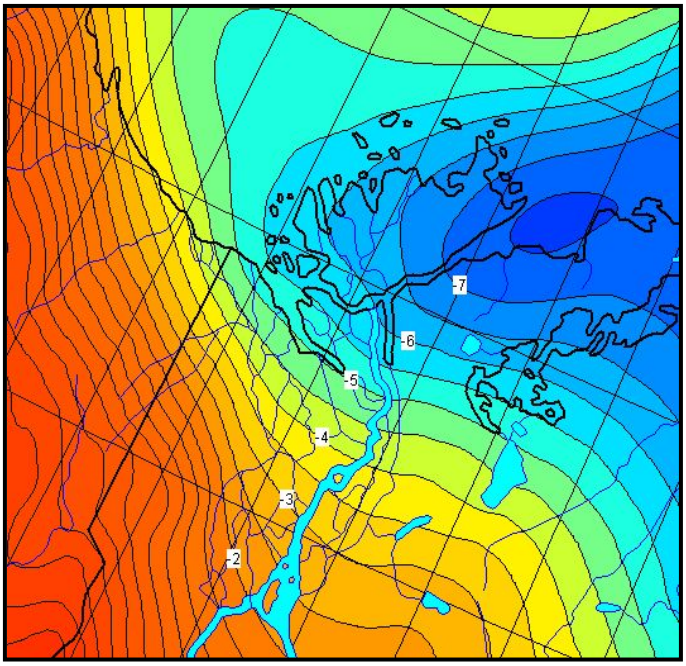

Figure 6: CGG05 geoid model for the Mackenzie Delta, NWT. (C.I.: 0.5 m)

On the other hand, the Canadian Gravimetric Geoid 2005 (CCG05) model agrees well with the Nov04 datum after removing a 80-cm bias (Figure A.6). The bias is irrelevant because the two datums (Nov04 and CGG05) do not represent the same equipotential surface by definition and include systematic errors. CGG05 corresponds to an equipotential surface representing global MSL. This surface is approximately 30 cm above local MSL in Rimouski. The standard deviation of the residuals ($h - H_{Nov04} - N_{CGG05}$) is 2.8 cm for the Delta region even though the geoid changes by 5 m across the region as shown in Figure 6. The statistic does not include the two stations in Tuktoyaktuk and two stations with large discrepancies. These larger discrepancies at stations 87T509 and 90T030 are probably due to vertical motion of the benchmarks between the leveling and GPS observation epochs. The separation between CGG05 and MSL (Δ_{CGG05}) is given by (station M039008):

$$\Delta_{CGG05} = h_{NAD83} - N_{CGG05/NAD83} - CD + Z_0. \quad = -2.767 - -7.135 - 5.113 + 0.364 = -0.381 \text{ m}.$$

For the two stations (M039007 and M039008) in Tuktoyaktuk, we notice from Figure A.6 that their residuals are smaller than the mean residuals of the other stations across the Delta by 11 and 14 cm, respectively. This step might indicate a problem with the leveling data or the geoid model. However, it would be unlikely that the error can be associated with the geoid model without affecting other stations in the Delta. It is also unlikely that it is a systematic error in the leveling because it would represent an error of 5 mm/km from station 80T7000, which is only 40 km west of Tuktoyaktuk (see Figure A.6). Most probably, there is a blunder in the leveling data somewhere along the 40 km stretch. This can be verified by re-leveling the segment (expensive and laborious) or by conducting a GPS survey on more benchmarks between the two stations. If there is a blunder and the local benchmarks in Tuktoyaktuk are tied to MSL, it would mean that

all heights in the Delta are actually higher than MSL in Tuktoyaktuk by 11 to 14 cm. Thus, comparing CGVD28 at the GPS stations of the 2004 campaign with CGG05 (Figure A.7) we notice again the systematic error in CGVD28 between Tsiigehtchic and Tuktoyaktuk and the step between stations 80T7000 (-23 cm) and station M039008 (-39 cm). As for Nov04, if the blunder is in the leveling data, CGVD28 heights should be higher by about 12 cm across the Delta. If we include the systematic error of 18 cm in CGVD28, the actual CGVD28 heights should be higher than MSL by an additional 30 cm when reaching Tsiigehtchic.

Finally, stations M039007 and M039008, which are just a few metres apart, can be considered as having identical geoid heights and the height difference measured by leveling between these two stations should have a precision at a few mm (1-3 mm). The observed difference of 3 cm between the two stations is probably due to GPS errors. It cannot be caused by instability of the stations because the leveling and GPS were conducted at the same epoch. Thus, when validating geoid models against GPS measurements at benchmarks, it is important to consider that the GPS error can be a few cm.

5. Recommendations

Is it possible to recommend an appropriate method for topographical measurements when such discrepancies exist between vertical datums within a relatively small region? Usually, for this size of area, the relative precision between the different datums is within a few cm, and certainly not at the decimeter level. The adoption of a geoid model as the official vertical datum for Canada, as proposed by the Height Modernization project, would eliminate the existing problems with CGVD28 and hybrid datums (e.g., HTv2.0). However, this modernization of the vertical datum would not be reality before 2009. In the meantime, projects are currently happening in the Delta region and a surveying method has to be recommended to surveyors.

The best approach is to work with ellipsoidal heights, i.e., using the ellipsoid as the vertical datum. They are accurate and precise measurements available in a cost-efficient manner when project areas extend over 5 to 10 km. The GPS heights are the original measurements and those to safeguard for future use such as for local crustal deformation. However, ellipsoidal heights alone cannot be used for water management because the geoid can change significantly within a project area. For example, the geoid changes by 2.5 m along the coastline between Tuktoyaktuk and the Yukon/NWT border. When GPS measurements are well established and secured, they can be easily transformed to heights above MSL by subtracting the geoid height. If more accurate geoid heights are available later on, new heights above mean sea level can be determined easily by retrieving the ellipsoidal heights. The key element is to document properly the geoid model used for the determination of the heights above MSL.

If the highest precision is required for proper water management, the ellipsoidal heights should be corrected using the latest geoid model, which is currently CGG05. It will allow the highest precision for orthometric heights in the region. These heights can be related to MSL by measuring the separation (Δ_{Datum}) between the vertical datum (geoid) and MSL at tide gauges. Δ_{Datum} should be a constant for a local area. Thus, the actual height above mean sea level (H_{MSL}) can be determined by:

$$H_{MSL} = h_{RF} - N_{Datum/RF} - \Delta_{Datum,} \qquad (9)$$

where subscripts *RF* and *Datum* are the names of the reference frame (e.g., NAD83 (CSRS), ITRF) and datum (e.g., CGVD28, HTv2.0, CGG05), respectively.

If the ellipsoidal height must be converted to CGVD28, the Height Transformation (e.g., HTv2.0) is the most efficient technique and would give accuracy better than 5 cm in most region of southern Canada where the leveling network is dense with GPS sites co-located with benchmarks. However, the error in HT can reach the decimeter level or more where GPS measurements on benchmarks are sparse, where the transformation was realized using early GPS ellipsoidal heights (1980's and early 1990's), and where the benchmarks are unstable. For these regions, it is preferable for the surveyors to realize their own height transformation using GPS measurements on local benchmarks to estimate the bias (and tilt) between the geoid model and CGVD28. Software GPS-H, developed at and available from Natural Resources Canada, allows users to create their own height transformation. Again, it is important to document properly how the local transformation was done.

6. Conclusion

Surveyors are currently at the crossroads between the traditional leveling and modern space-based positioning technology for the determination of accurate and precise heights. The leveling is highly precise, but it is a laborious technique dependent on access and for which cost increases as a function of distance. GPS is a highly efficient and cost-effective positioning technique at most locations. Unfortunately, heights derived from either technique do not necessarily coincide because of systematic errors in their reference datum. In particular, CGVD28, which is a 1928 construct datum, contains several systematic errors making it incompatible with accurate geoid models developed today. Hybrid datums, which distort a geoid model to make it fit to an official datum realized by leveling, are only a band-aid solution that adds more confusion at times because they coincide neither with the official datum nor the geoid model in regions where benchmarks are sparse or inexistent (e.g., northern Canada).

Discrepancies between different datums are demonstrated for the region of the Mackenzie Delta, NWT. Precise heights are required in this low-lying area along the Beaufort Sea because river and storm-surge flooding are significant issues in relation to important economic activity and environmental concerns. CGVD28, geoid model CGG05 and hybrid datum HTv2.0 disagree at the level of a few decimeters in the Delta. CGVD28 has systematic errors that accumulate to 18 cm from Tsiigehtchic to Tuktoyaktuk because of constraints imposed in its adjustment. Furthermore, the leveling data, west of Tuktoyaktuk, may contain a blunder that would increase the error in CGVD28 by an additional 10-15 cm making CGVD28 heights too low in Tsiigehtchic by about 30 cm with respect to MSL at the tide gauge in Tuktoyaktuk. HTv2.0, which was realized in 2001, is not appropriate for the Delta region because it is based on only two GPS ellipsoidal heights collocated with benchmarks in Inuvik and Tsciigehtchic. These measurements date back to 1987 when GPS accuracy was at the decimeter level. Furthermore, the measurements were too sparse to investigate the stability of the observed benchmarks.

The best approach to determine accurate heights above MSL is through the combination of GPS and an accurate gravimetric geoid model (CGG05). Even though geoid and MSL do not coincide, the separation between the two surfaces can be measured at the reference markers of tide gauges. However, the fundamental values are the ellipsoidal heights, which are accurate measurements in reference to a stable reference surface: an ellipsoid. The orthometric height is derived by subtracting a geoid height that will improve continuously over the next few years. Thus, it is important to secure GPS information and document properly the method used to transform ellipsoidal heights to heights above mean sea level.

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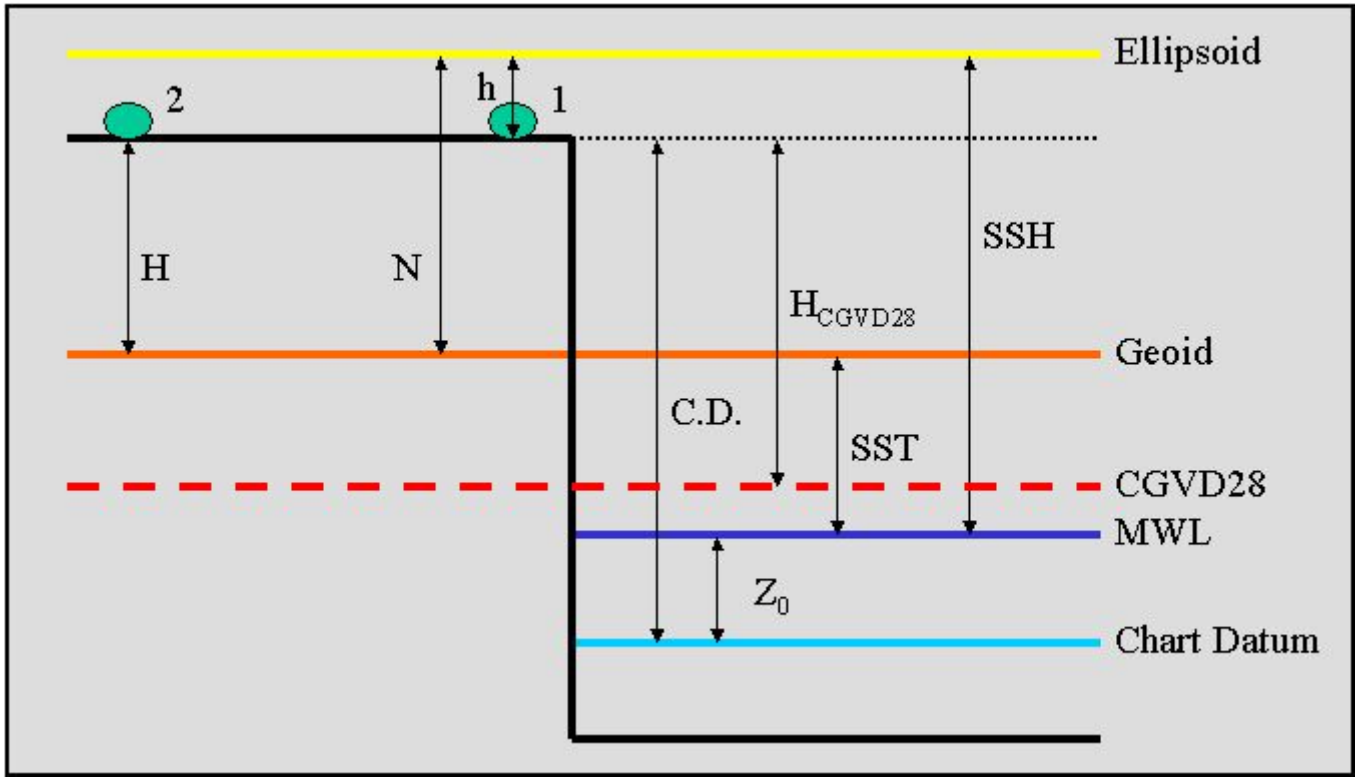

Figure A.1: Illustration representing the relation of the different vertical datums in Tuktoyaktuk, NWT. Points 1 and 2 represent two stations along the coast. H is an orthometric height; h is an ellipsoidal height (negative in this case); N is the geoid height (negative in this case); C.D. is the height of a station above chart datum; Z_0 is the height of the mean water level (MWL) above the chart datum; H_{CGVD28} is the "orthometric" height above CGVD28; SST is the sea surface topography (height of sea surface above the geoid); and SSH is the Sea Surface Height (height of the sea surface above the ellipsoid). The ellipsoid, geoid, CGVD28, MWL and chart datum are not parallel surfaces in reality.

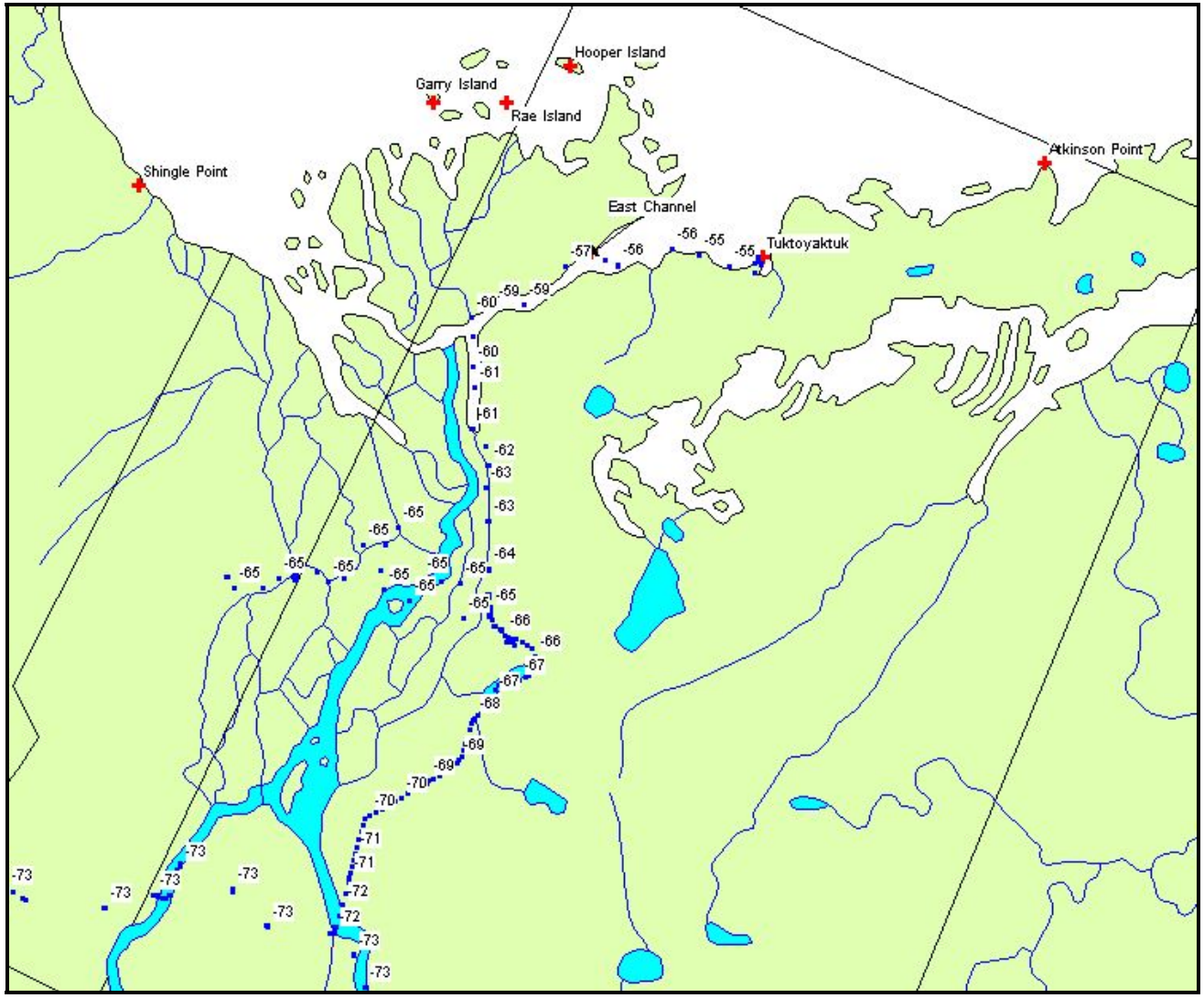

Figure A.2: Difference between the leveling datums of Nov04 and CGVD28 (CGVD28 – Nov04) for the region of the Mackenzie Delta.

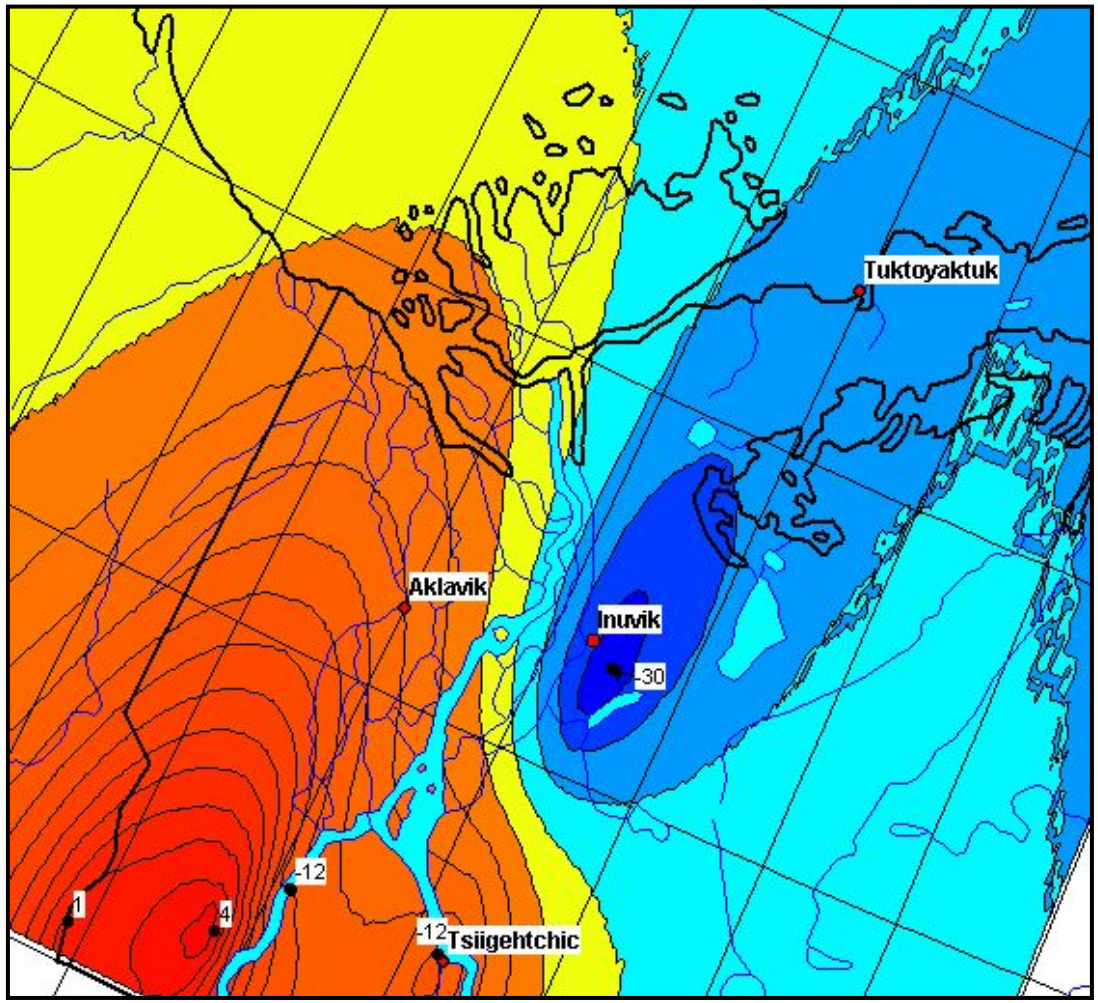

Figure A.3: Distortions applied to CGG2000 to represent CGVD28 in the Mackenzie Delta (C.I.: 2 cm). The black points represent GPS stations on benchmarks available for the realization of HTv2.0. The values, in cm, indicate h-H-N where h is the ellipsoidal heights, H is the CGVD28 heights and N is the CGG2000 geoid height.

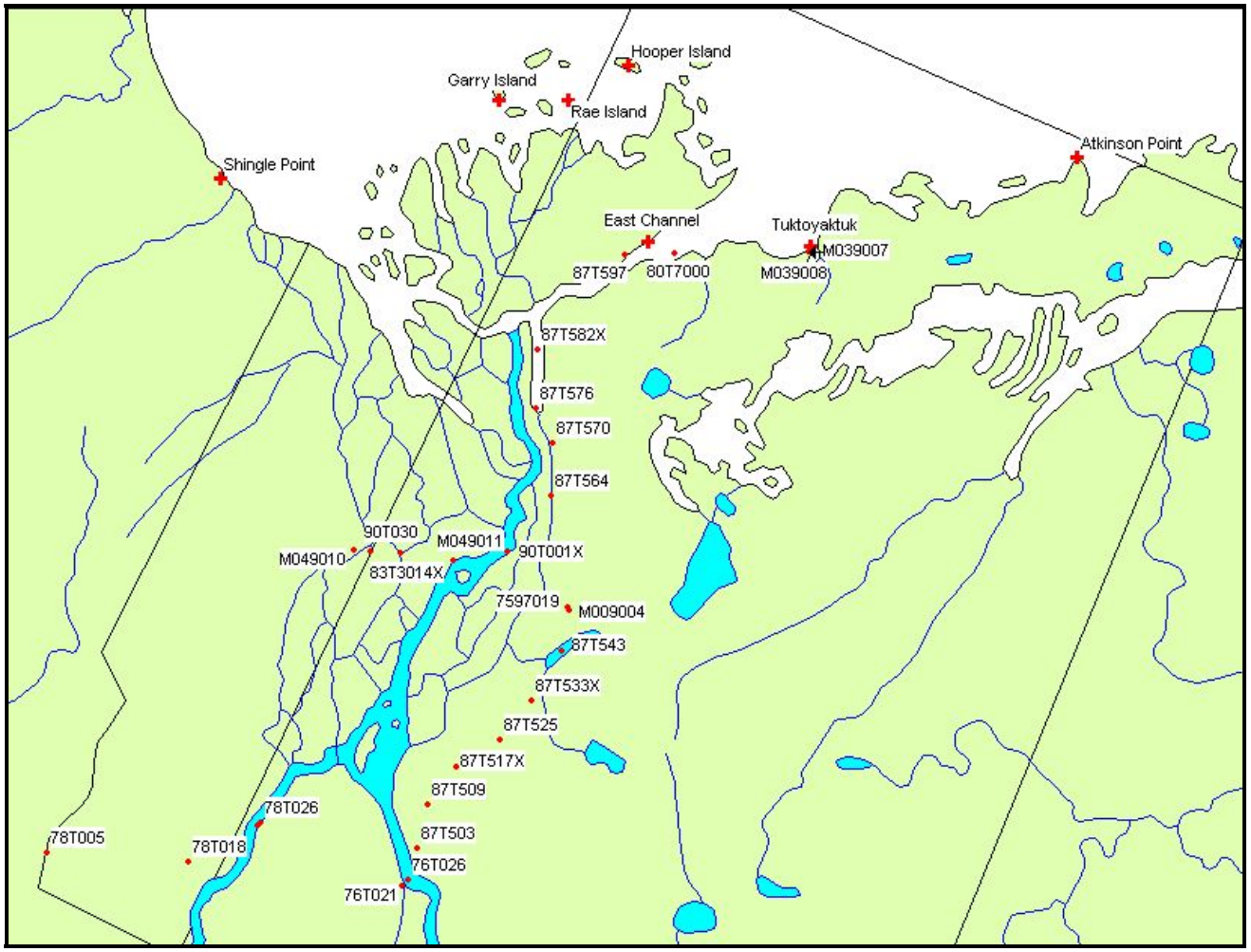

Figure A.4: Identification of the stations observed during the 2004 GPS campaign in the Mackenzie Delta, NWT.

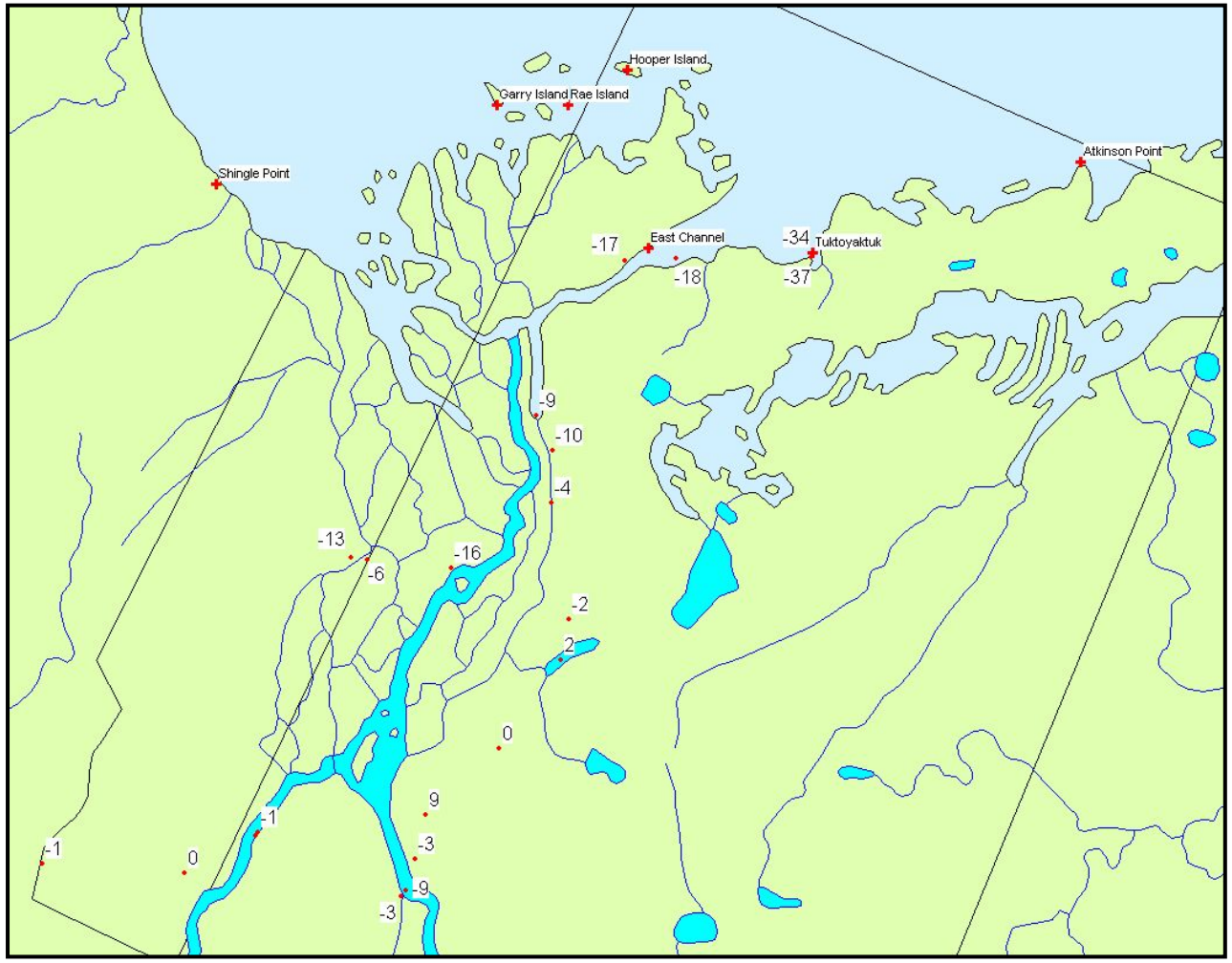

Figure A.5: Difference between the leveling datum of CGVD28 and height transformation datum of HTv2.0 at benchmarks in the region of the Mackenzie Delta. The discrepancy is determined by $h - H - N^*$ where h is the ellipsoidal height in ITRF2000 measured by GPS, H is the CGVD28 orthometric height and N^* is the "geoid height", i.e., height transformation (HTv2.0). The discrepancies are expressed in cm.

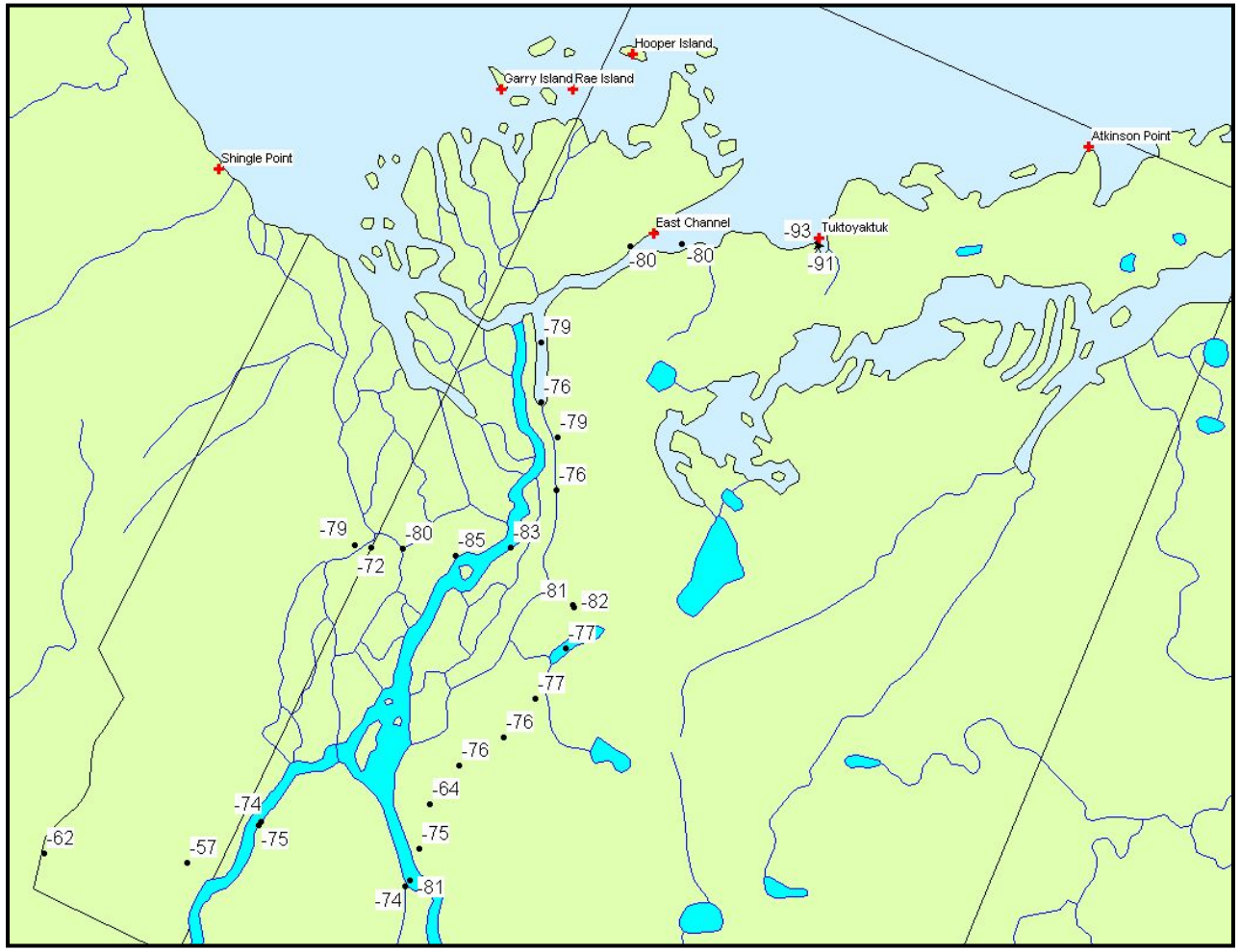

Figure A.6: Difference between the leveling datum of Nov04 and geoid datum of CGG05 at benchmarks in the region of the Mackenzie Delta. The discrepancy is determined by $h - H - N$ where h is the ellipsoidal height in ITRF2000 measured by GPS, H is the orthometric height (Nov04) and N is the geoid height (CGG05). The discrepancies are expressed in cm.

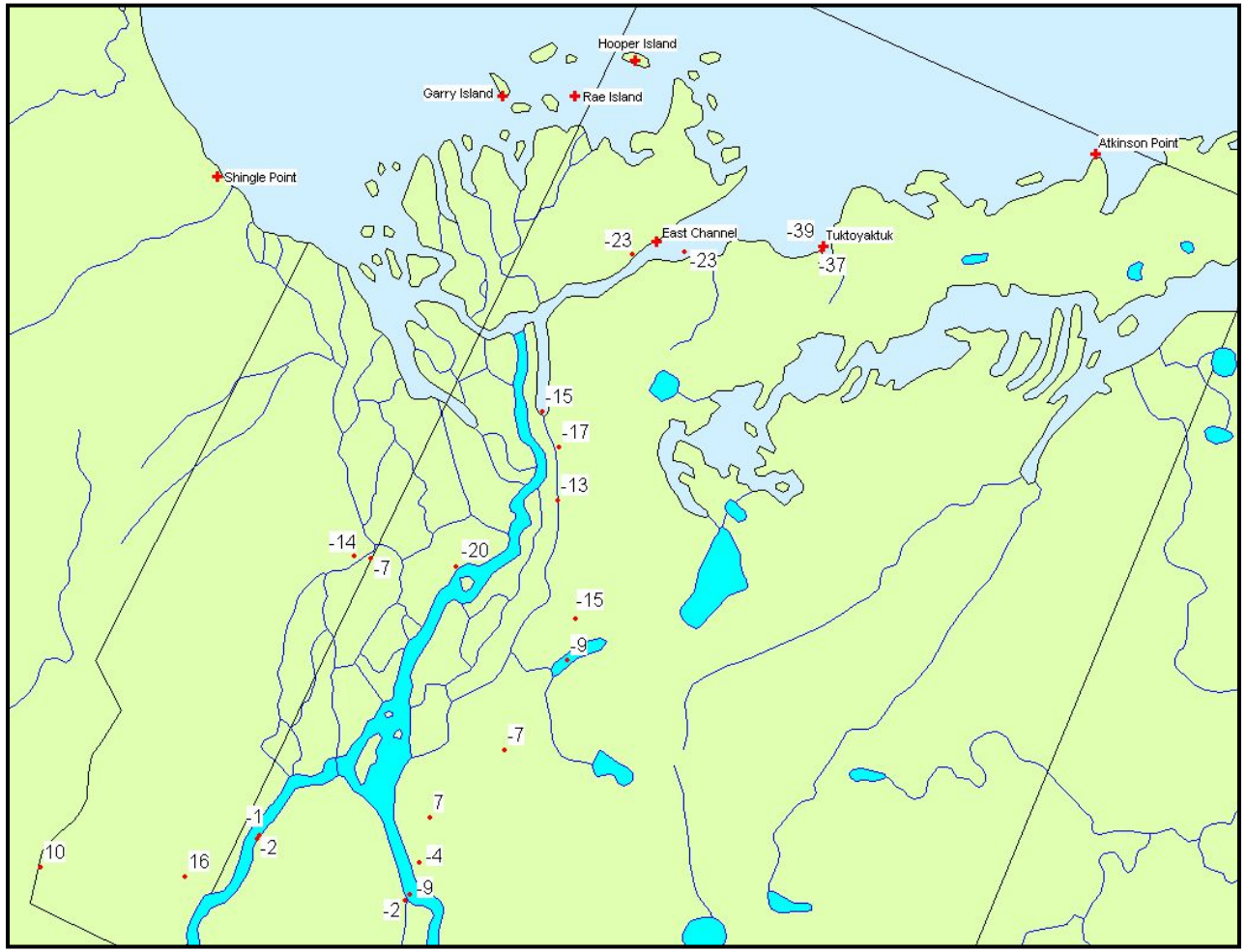

Figure A.7: Difference between the leveling datum of CGVD28 and geoid datum of CGG05 at benchmarks in the region of the Mackenzie Delta. The discrepancy is determined by $h - H - N$ where h is the ellipsoidal height in ITRF2000 measured by GPS, H is the CGVD28 orthometric height and N is the geoid height (CGG05). The discrepancies are expressed in cm.